Definition of De Morgan's Laws:

The complement of the union of two sets is equal to the intersection of their complements and the complement of the intersection of 2 sets is equal to the union of their complements. These are called De Morgan's laws.

For any two finite sets A and B,

(i) \((A \cup B)' = A' \cap B'\) (which is a De Morgans law of union).

(ii) \((A \cap B)' = A' \cup B'\) (which is a De Morgans law of intersection).

De Morgan's First Law:

\((A \cup B)' = A' \cap B'\)

The first law states that the complement of the union of two sets is the intersection of the complements.

Proof:

Let \(P = (A \cup B)'\) and \(Q = A' \cap B'\)

Let \(x\) be an arbitrary element of \(P\) then \(x \in P = x \in (A \cup B)\)

\(= x \notin (A \cup B)\)

\(= x \in A'\) and \(x \in B'\)

\(= x \in A' \cap B'\)

\(= x \in Q\)

Therefore, \(P \subseteq Q\) ....... (i)

Again, let \(y\) be an arbitrary element of \(Q\) then \(y \in Q = y \in A' \cap B'\)

Again, let \(y\) be an arbitrary element of \(Q\) then \(y \in Q = y \in A' \cap B'\)

\(= y \in A'\) and \(y \in B'\)

\(= y \in A'\) and \(y \in A \cup B\)

\(= y \in (A \cup B)'\)

\(= y \in P\)

Therefore, \(Q \subseteq P\) ....... (ii)

Now combine (i) and (ii) we get; \(P = Q\) i.e. \((A \cup B)' = A' \cap B'\)
Demorgan's Second Law

\[(A \cap B)' = A' \cup B'\]

The second law states that the complement of the intersection of two sets is the union of the complements.

Let \(M = (A \cap B)'\) and \(N = A' \cup B'\)

Let \(x\) be an arbitrary element of \(M\) then \(x \in M = x \in (A \cap B)'

\[= x \notin (A \cap B)\]

\[= x \notin A \lor x \notin B\]

\[= x \in A' \lor x \in B'\]

\[= x \in A' \cup B'\]

\[= x \in N\]

Therefore, \(M \subseteq N \quad \text{(i)}\)

Again, let \(y\) be an arbitrary element of \(N\) then \(y \in N = y \in A' \cup B'\)

\[= y \in A' \lor y \in B'\]

\[= y \notin A \lor y \notin B\]

\[= y \notin (A \cap B)\]

\[= y \in (A \cap B)'\]

\[= y \in M\]

Therefore, \(N \subseteq M \quad \text{(ii)}\)

Now combine (i) and (ii) we get; \(M = N\) i.e. \((A \cap B)' = A' \cup B'\)